Earth-tide as parameter of crustal motion correction for SLR station displacement

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ABSTRACT

A new Satellite Laser Ranging (SLR) analysis program is being developed at Hartebeesthoek Radio Astronomy Observatory (HartRAO) that can model satellite orbits with high accuracy utilising advanced modelling and SLR data. We introduce the background to SLR analysis to an interdisciplinary audience and present the results of processing one month of LAGEOS-1 and LAGEOS-2 data in combined and separate solutions. The inclusion of Earth-tide modelling is shown to be necessary to reduce range bias and to minimise observed minus computed (O-C) residuals. The combined solution shows a reduction in range bias when allowance is made for the SLR station displacement vector due to solid Earth-tide and the effect of the Earth-tide on the static gravity field. In order to make the comparison it is necessary to set the a-priori estimation error of unmodelled forces to a low value as it tends to absorb range biases and improve O-C residuals if included as an additional solve-for parameter. A comparison between the SLR station position perturbation resulting from solid Earth-tide and range bias indicates a correlation, which probably results from an overestimate of the Earth-tide vector and indicates that SLR will be able to differentiate between and evaluate different Earth-tide models. These are preliminary results and further improvement is envisaged due to the planned inclusion of additional advanced modelling procedures. Applications of this software will include velocity field determinations for crustal dynamics studies.

Introduction

The Space Geodesy Programme of HartRAO produces data that are multi-disciplinary in nature and are used in diverse research topics such as geophysics, plate tectonics, neotectonics, gravity, oceanography, meteorology, space science and fundamental physics. Within the framework of Inkaba yeAfrica, the research initiative proposed by HartRAO reflects the objectives of a multi-disciplinary, long-term earth science project with special emphasis on multi-techniques and capacity building in the southern Africa region.

A multitude of forces continuously act upon the solid Earth. External forces result from the gravitational attraction of the Sun, Moon, and planets. Surface forces are due to the action of the atmosphere and oceans. Internal forces are due to mantle convection, tectonic motions and coupling between the mantle and both the fluid outer core and the solid inner core. The solid Earth reacts to these forces by undergoing rotational accelerations, mass displacements, and continuous deformation, which in turn affects the position of a space geodetic system located on the surface of the Earth. This paper will address and quantify the effect that non-inclusion of station displacement due to the solid Earth-tide has on the O-C residuals and range bias for a selected station (Yarragadee, 7090).

The present lunar and artificial satellite laser ranging (LLR/SLR) tracking network, through the aegis of the International Laser Ranging Service (ILRS), is responsible for monitoring an increasingly large number of Earth-orbiting satellites. HartRAO supports the ILRS through operating MOBLAS6 as part of the NASA SLR network. A host of scientific disciplines are being investigated using these data. These include the realisation, maintenance, and improvement of the International Terrestrial Reference Frame (ITRF) and monitoring the 3-dimensional deformation of the solid Earth.

SLR is a space geodetic technique whereby a short laser pulse is transmitted to a satellite equipped with a suitable corner cube reflector that reflects the incoming laser light back to the SLR. A telescope (part of the SLR system) collects the returning photons and by utilising a precise clock, the round-trip time from pulse transmission to pulse reception can be determined. Several satellites have been launched for specific geodetic objectives such as LAGEOS-1 and LAGEOS-2 (LAser GEOdynamics Satellite). These satellites have a small area-to-mass ratio which minimizes the effects of non-gravitational forces such as experienced from solar pressure. SLR accuracy and orbit determination techniques can fit a three-day arc of LAGEOS data to a precision of 1 to 2 cms. LAGEOS-1 was launched on May 4, 1976 and LAGEOS-2 on October 22, 1992. Both satellites have a diameter of 60 cm and are spherical in shape; their surfaces are covered with 426 corner cube reflectors.

SLR analysis program

Currently we are developing an SLR analysis program that processes normal point data (data that are generated from raw data to statistically improve its accuracy). Data are obtained by ILRS SLR stations; see for instance Pearlman *et al.* (2002).

One of the objectives of the SLR analysis program is to determine an accurate position of the centre of mass of the satellite tracked by SLR at the epoch of the normal

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point derived from SLR data. Using these positions, the orbit or orbital arcs can be determined or improved. Once an accurate orbit has been determined, a strategy can be developed to determine certain parameters such as the position of the SLR station or over a longer time period (2+ years) the velocity of the SLR station. Other parameters can also be solved for, including the coefficient of reflection or atmospheric drag. The station positions are expressed in an appropriate Earth-fixed reference frame such as the International Terrestrial Reference Frame (ITRF) and earth orientation parameters used for necessary transformations are provided by the International Earth Rotation Service (IERS).

Several forces need to be taken into account when determining the orbit of the satellite.

The gravitational forces perturbing the orbit of the satellite consist of:

- Earth's geopotential
- Solid earth tides
- Ocean tides
- Planetary third-body perturbations (Sun, Moon and planets)
- Relativistic accelerations
- Atmospheric tide

The non-gravitational forces consist of:

- Atmospheric drag
- Solar radiation pressure
- Earth radiation pressure
- Thermal radiation acceleration

Perturbed equations of motion

The satellite equations of motion can be expressed as (Tapley *et al.* 2004)

$$\ddot{r} = -\frac{\mu \mathbf{r}}{r^3} + \mathbf{f}$$

where **r** is the position vector of the satellite $\mu = G(M_1+M_2) \cong GM_1$ (G is the gravitational constant M_1 and M_2 are the masses of the Earth and satellite respectively), r is the distance between the satellite and the earth centre of mass. The orbit is constantly perturbed by force (centre of mass),

$$\mathbf{f} = \mathbf{f}_{\mathrm{NS}} + \mathbf{f}_{\mathrm{3B}} + \mathbf{f}_{\mathrm{g}} + \mathbf{f}_{\mathrm{Drag}} + \mathbf{f}_{\mathrm{SRP}} + \mathbf{f}_{\mathrm{ERP}} + \mathbf{f}_{\mathrm{Other}}$$

where $\mathbf{f}_{NS}$ is the contribution of the mass distribution of the Earth, $\mathbf{f}_{3B}$ is the perturbing force contribution from the Sun, Moon and planets, $\mathbf{f}_g$ represents contributions resulting from general relativity, $\mathbf{f}_{DRAG}$ is the negative acceleration caused by atmospheric drag, $\mathbf{f}_{SRP}$ represents the solar radiation force contribution, $\mathbf{f}_{ERP}$ the Earth radiation pressure and $\mathbf{f}_{Other}$ the forces that are not modelled. The modelling of these forces will not be described here in more detail. For LAGEOS satellites we use a concentric annulus model developed to improve solar radiation modelling. The complexity of the model can be pre-set; for this work the satellite was modelled by using ten concentric bands to better approximate the spherical shape of the satellite instead of using a flat plane for the area projected towards the Sun. Varying incident angle compensation improves the determination of the coefficient of reflection of the satellite (LAGEOS-1 and -2 = 1.12).

Corrections to SLR range measurements

The two-way range (uplink and downlink) is based on the time-of-flight principle and is computed using an iterative procedure to determine the surface time of arrival. Range bias results from the sum of all systematic instrumental deviations and environmental errors (Schillak, 2004) and needs to be taken into account. The range bias is calculated by taking the average of the O-C residuals, the range bias is then re-introduced into the least-squares algorithm for orbit calculations as an additional correction and a complete re-adjustment is made. Different strategies exist to solve for range bias and its accuracy is affected by the quality of the orbit determination and the quantity and quality of the normal points used for the orbit calculation.

At optical wavelengths the troposphere is dispersive and a correction for an additional delay due to the troposphere must be made. An often-used correction for atmospheric delay is given by Marini and Murray (1973). The HartRAO software utilises a modification of this approach as implemented by Mendes *et al.* (2002), which implements newly derived mapping functions for optical wavelengths, using a large database of ray tracing radiosonde profiles. These functions are optimised for a wavelength of 0.532 μm and are valid for elevation angles greater than three degrees, if one neglects the contribution of horizontal refractivity gradients. Typical SLR data processing rejects data captured below 20 degrees and our software makes provision for a variable setting for rejection of data below any selected elevation setting. These corrections can be several metres and the delay is dependent on the wavelength of the laser signal.

Additional corrections could be necessary to refer the SLR system to a benchmark tied to bedrock; this is normally added as an offset. Some systems with X-Y mounts where the axes intersect may not require such an offset correction. In addition, technique dependent systematic errors that are linked to the hardware and software of the SLR system will affect the range bias.

Coordinate and reference system

The station coordinates and the initial coordinates of the satellites are ITRF coordinates and the satellite a-priori coordinates are provided by the ILRS in the consolidated prediction format (CPF). During processing, both satellite and SLR station position vectors are transformed to a non-rotating (inertial) frame, the International Celestial Reference Frame (ICRF).

The ICRF is a geocentric inertial coordinate system, defined by the mean equator and vernal equinox at Julian epoch 2000.0 and is termed the J2000 system. To account for the gravitational perturbations on the satellite orbit by the Sun, Moon and planets, we utilise the Jet Propulsion Laboratory (JPL) DE-405 planetary ephemeris (Standish, 1998), which is based on the ICRF inertial coordinate system. These coordinates have been converted from barycentric inertial to geocentric inertial. The SLR station coordinates are transformed to the ICRF reference frame by taking into account polar motion of the Earth, precession and nutation, and UT1 transformation. These data are obtained from the International Earth Rotation Service (IERS) and we utilise Bulletin B, with all values interpolated via polynomial fits to the epoch of SLR measurement. The software conforms to the IERS96 conventions as far as precession and nutation of the Earth's polar motion is concerned which specifies the 1976 International Astronomical Union (IAU) precession (Lieske *et al.*, 1977; Lieske, 1979) and the 1980 IAU nutation formula (Wahr, 1981; Seidelmann, 1982). An additional correction is added which is derived from VLBI analysis (Herring *et al.*, 1991). UT1-UTC values are as provided by IERS Bulletin B.

Earth tide effects on station position and variations of the static gravity field

The SLR station position is continuously disturbed by variations in the position of the Earth's crust due to horizontal and vertical displacements caused by tidal perturbations and tectonic plate velocity. Tectonic plate motion can be taken into account by calculating the plate velocity using ITRF station velocities and adjusting the station position in the ITRF to the epoch of the SLR observations. The gravitation of the Sun and Moon exerts a force on the Earth, which leads to a time-varying deformation of the Earth. These small periodic deformations of the Earth are termed solid Earth tides, with amplitudes ranging up to 35 cm. These Earth-tides consequently affect the gravity field of the Earth, which in turn affects the motion of the satellites.

We are utilising the Sotid library (Petrov, 2005) in our software for computing station displacements due to solid Earth-tides. This library uses the frequency domain approach and the HW95 (Hartmann and Wenzel, 1995) expansion of the tide-generating potential. Love number h_{nm} and Shida number l_{nm} characterise site displacements caused by tides of spherical harmonic degree and order *nm*. The Love number is related to vertical displacement and the Shida number to horizontal displacement. All generalised Love numbers of the second degree were used. The Sotid library supports several Love number models of which we used the model of P. Mathews submitted to IERS Conventions, 2000, revision of 2001.10.10 as implemented by Petrov (2005). All orders of tidal waves of the second degree were taken into account, *i.e.* a combination of zonal, diurnal and semi-diurnal orders. Third degree tides were included using the model of Mathews, Dehant and Gipson (1997). The computation of Earth-tide and Ocean-tide loading (which affects the station position by 1-2 cm, a factor of ten less than Earth-tide) is quite complex and we will only give a brief description here of the gravitational potential.

The gravitational field of the Moon or Sun which has mass M implies that at point P on the surface of the Earth a potential U exists, where

$$U = \frac{GM}{|\mathbf{s}-\mathbf{R}|} + \frac{1}{2}n^2 d^2$$

Here **R** is the geocentric coordinates of P and **s** is those of the tide generating body, d is the distance of P to an axis through the system's centre of mass and n the mean motion of the body about this axis. Following Montenbruck and Gill (2000), the denominator of the expression describing U can be expanded to

$$\frac{1}{|\mathbf{s}-\mathbf{R}|} \approx \frac{1}{s}\left(1+\frac{R}{s}\right)\cos\gamma - \frac{1}{2}\frac{R^2}{s^2} + \frac{3}{2}\frac{R^2}{s^2}\cos^2\gamma$$

as $s >> R$ for the Sun and the Moon. The angle between **s** and **R** is γ. The distance of the point P to the axis through the system's centre of mass can be written as

$$\begin{aligned} d^2 &= d_c^2 + R^2\cos^2\phi\cos(\Delta\lambda) \\ &= d_c^2 + R^2\cos^2\phi - 2d_c\cos\gamma \end{aligned}$$

Here $d_c = Ms/(M+M_\oplus)$ describes the geocentric distance of the centre of mass of the system where ϕ is the geocentric latitude of the centre of mass. The difference of the East longitudes of P and the perturbing body is given by $\Delta\lambda$. Using the relation $n^2s^3 = G(M+M_\oplus)$ (see Bertotti and Farinella, 1990), the potential can be written as

$$U = \frac{GM}{s}\left(1+\frac{1}{2}\frac{M}{M+M_\oplus}\right) + \frac{GMR^2}{2s^3}\left(3\cos^2\gamma - 1\right) + \frac{n^2R^2}{2}\cos^2\phi$$

The third term adds a small permanent equatorial bulge to the Earth that is smaller than that due to the rotation of the Earth as $n^2 = \omega_\oplus^2$. The first term is constant whereas the second term (U_2) is a second-order zonal harmonic whose amplitude is proportional to GM/s^3 so that the tides due to the Moon are about twice as strong as those due to the Sun. This tidal potential basically describes an elastic deformation of the Earth. Deviations from this simplistic elastic response are a result of friction (which causes phase lags of the tidal bulge relative to the position of the Sun and Moon) and the rate-dependent behaviour of the Earth's oceans. The tidal-induced gravity potential has many different periods as the angle γ varies as the position of the Sun and Moon changes with respect to the rotating Earth.

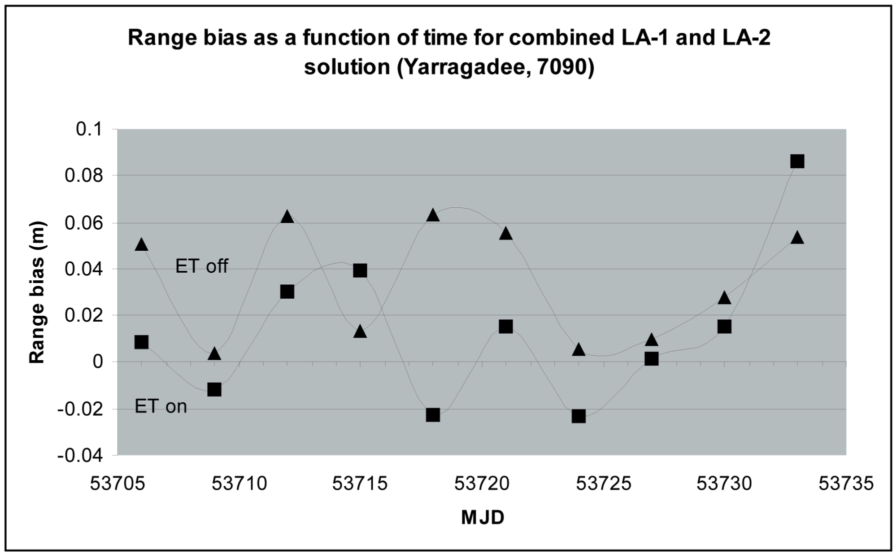

Figure 1. Range bias for the combined solution of LAGEOS-1 and LAGEOS-2 indicating a reduction in range bias as a result of including Earth-tide modelling. Perturbations due to Earth-tide effects on the static gravity field was disabled.

Changes in the eccentricity of the orbits of the Sun and Moon result in monthly and annual periods. Detailed information concerning the calculation of station displacement due to the solid Earth tide can be found in IERS (2003).

In addition to the station position variation caused by the solid Earth tide, this tide (as well as the ocean and pole tides) causes a change in the Earth's gravitational potential and therefore adds to the accelerations acting on the satellite. The Earth's gravity field is therefore not static, but contains small periodic variations, which will affect the motion of the satellites. These perturbations can be derived using an expansion of the tidal-induced gravity potential utilising spherical harmonics in an analogous way as for the previously discussed static gravity field. Detailed descriptions of the calculations of the variations in the standard geopotential coefficients C_{nm} and S_{nm} due to solid Earth tide; solid Pole tide and the effect of the ocean tides can be found in IERS (2003). For practical purposes (Montenbruck and Gill, 2000) these corrections to the unnormalized geopotential coefficients can be calculated using

$$\begin{Bmatrix}\Delta C_{nm}\\ \Delta S_{nm}\end{Bmatrix} = 4k_n \left(\frac{GM}{GM_\oplus}\right)\left(\frac{R_\oplus}{s}\right)^{n+1}\sqrt{\frac{(n+2)(n-m)!^3}{(n+m)!^3}}\,P_{nm}(\sin\phi)\begin{Bmatrix}\cos(m\lambda)\\ \sin(m\lambda)\end{Bmatrix}$$

for the Sun and Moon respectively. Here k_n are the Love numbers of degree n. The Earth-fixed latitude and longitude of the disturbing body are given by ϕ and λ respectively. This acceleration of the satellite decreases with $1/r^4$ and is especially important for low earth orbiters. For the case of LAGEOS in our calculations during this work the accelerations were of the order 10^{-8}. The contribution by ocean tides is about one order of magnitude smaller and at this stage is not considered in our software application. The acceleration caused by solid Earth tides when considered as an indirect gravitational effect of the Sun and Moon can also conveniently be calculated as (Rizos and Stolz, 1985)

$$\ddot{\vec{r}}_e = \frac{k_2}{2}\frac{Gm_d}{r_d^3}\frac{a_e^5}{r^4}(3-15\cos^2\theta\frac{\vec{r}}{r}+6\cos\theta\frac{\vec{r}_d}{r_d})$$

where m_d is the mass of the disturbing body (Sun, Moon), $\vec{r}_d$ the geocentric position vector of the disturbing body, θ the angle between the geocentric vector $\vec{r}$ and $\vec{r}_d$, and k_2 is the Love number which describes the elasticity of the Earth's body (0.299). For more accurate solutions (especially when low Earth orbiters are involved) the recommendations of the IERS (2003) will be more appropriate.

Ocean loading effects on station position and variations of the static gravity field

Ocean tide loading deforms the Earth's crust due to the additional weight of the ocean tides. This deformation leads to three dimensional station position variations. Similar to Earth-tide the ocean tides are caused by the gravitational pull of the Sun and Moon so that in effect the ocean tides can be described as a sum of several ocean tides of different periods. Typically the 11 harmonics with the largest amplitudes are used to compute the ocean tide loading. The amplitudes of the ocean loading tides as they affect the station position and the static gravity field are about one order of magnitude smaller than those of the solid Earth tides. In our calculations during this work for the SLR station Yarragadee in Australia, typical ocean loading site displacement vectors are between 2 and 14 mm. The software uses coefficients computed according to Scherneck (1991). These are available from

Table 1. lists the analysis strategy, conventions and programme settings that were adopted:

Celestial reference frame	J2000
Terrestrial reference frame:	ITRF2000 epoch 1997.0
Solar, lunar and planetary ephemerides for 3rd body gravitational perturbation	JPL DE405 (Standish, 1998.)
Pole-tide correction (station position)	IERS 2003
Pole-tide acceleration of satellite	Not implemented
Relativity (space-time curvature)	IERS 2003
Earth–tide correction (station position)	Petrov 2005
Earth-tide acceleration of satellite	(Rizos and Stolz, 1985)
Ocean loading correction (station position)	Scherneck, 1991
Atmospheric loading	Not implemented
Definition of origin	Geocentric
Gravity model	JGM-3 (20x20) (Tapley *et al.*1996)
LAGEOS-2 model	Concentric annulus x 10
Reference epoch	1997.0
Tectonic plate model	ITRF2000 velocity field
Earth orientation	a-priori Earth orientation parameters and UTC-UT1 values as per IERS Bulletin B extrapolated to observation epoch
A priori precession model	IAU(1976) (Lieske, 1976)
A priori nutation model	IAU(1980) (Seidelmann, 1980) and dPsi and dEpsilon corrections (Herring *et al.* 1991) from IERS Bulletin B
O-C outlier rejection	> 1 sigma or 10 cm
Data rejection	<10 degrees elevation
Range bias	Enabled
Time bias	Disabled
Satellite centre-of-mass	251 mm, ILRS standard value (Otsubo and Appleby, 2003)

Reference frames, constants and models used in the analysis of the SLR data.

http://www.oso.chalmers.se/loading. We use a recent ocean tide map model (GOT00.2, a TOPEX/POSEIDON derived solution) by Ray (1999). GOT99.2b and GOT00.2 are long wavelength adjustments of FES94.1 (a pure hydrodynamic model tuned to fit tide gauges globally) using TOPEX/POSEIDON data and are given on a 0.5 by 0.5 degree grid. To compute the three-dimensional SLR site displacement we follow IERS (2003) recommendations. If Δc denotes the displacement component (radial, west, south) at a time t for a particular site, then (IERS, 2003)

$$\Delta c = \sum_j f_j A_{cj} \cos(\omega_j t + \chi_j + u_j - \phi_{cj})$$

with

$$a_{cj}\cos\phi_{cj} = H_j\left[\frac{A_{ck}\cos\phi_{ck}}{H_k}(1-p) + \frac{A_{c,k+1}\cos\phi_{c,k+1}}{\bar{H}_{k+1}}p\right],$$
$$a_{cj}\sin\phi_{cj} = H_j\left[\frac{A_{ck}\sin\phi_{ck}}{H_k}(1-p) + \frac{A_{c,k+1}\sin\phi_{c,k+1}}{\bar{H}_{k+1}}p\right]$$

where f_j and u_j depend on the lunar node's longitude, ω_j is the tidal angular velocity, χ_j the astronomical argument at time $t = 0^b$. The amplitudes A_{ck} and phases $\phi_{ck,}$ $1 \leq k \leq 11$ are taken from the GOT00.2 model.

Consequently we convert the radial, west and south components of the displacement to Δx, Δy, Δz Earth centred, Earth fixed components, which are then added to the station position at the SLR measurement epoch.

The effect of ocean tides on the static gravity field is not yet implemented in our software, but complete descriptions of their incorporation as variations in the standard geopotential coefficients can be found in IERS (2003).

Observations and data analysis

Data from the Australian SLR station Yarragadee were processed for the month of 1 to 30 December 2005. Due to the developmental stage of our software it was not feasible to process longer sets of data and one month is adequate to test the Earth-tide implementation. The Yarragadee station is the best in the SLR network as far as data quantity is concerned and continuously provides high quality data. For the period processed there were 1721 normal points available, slightly more for LAGEOS-2 (891) than for LAGEOS-1 (830). We decided to process both LAGEOS-1 and LAGEOS-2 in a simultaneous solution yielding a combined RMS and mean value for the O-C residuals as well as a combined range bias determination. This means that two orbits are solved for simultaneously, one for each satellite, however the location where the solid Earth-tide affects the station position (Yarragadee) is the same for each satellite orbit. Combining the two orbital solutions increases the available data and reduces biases that could result from a single orbit. Both satellites were also processed independently to evaluate their orbital solutions for comparison purposes.

Data of one month were processed in steps of three-day arcs, firstly with Earth-tide modelling enabled and secondly disabled. We disabled (by setting its a-priori standard deviation to a very low value) a solve-for parameter, which models unknown perturbations due to solar pressure acceleration. Small, unmodelled forces are typically accounted for using the concept of empirical accelerations (all models are limited by uncertainties arising from material properties, varying satellite

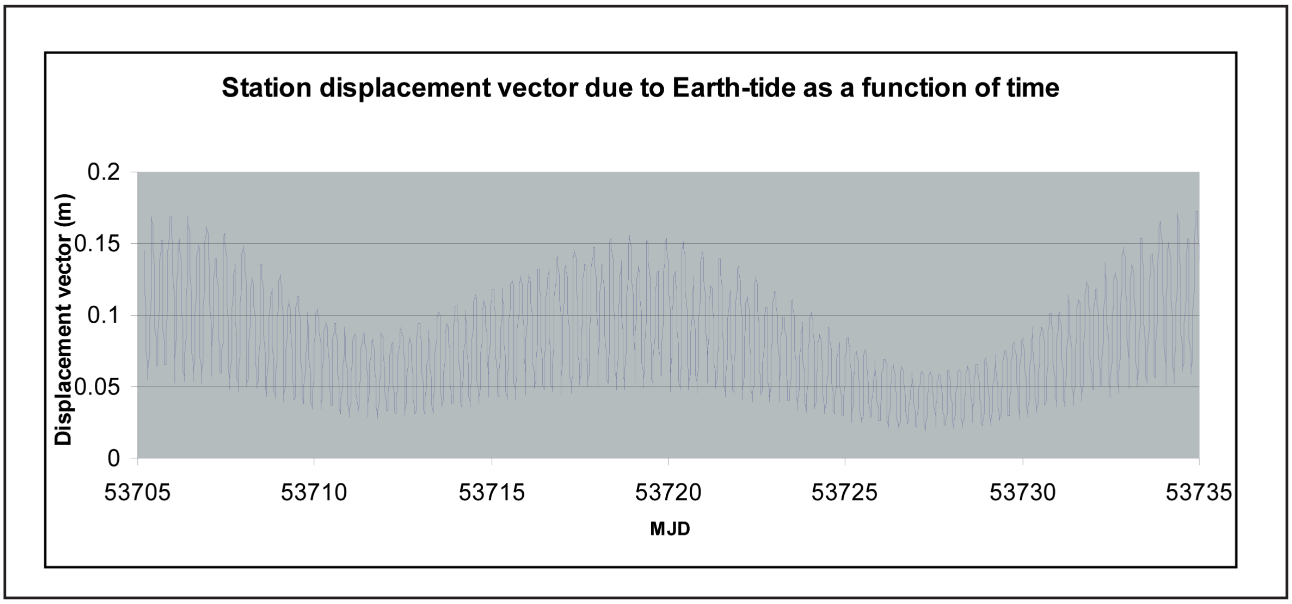

Figure 2. Position displacement of the SLR station Yarragadee (Australia)due to Earth-tide indicating diurnal sub-and longer periods due to the gravitational potential of the Sun and Moon.

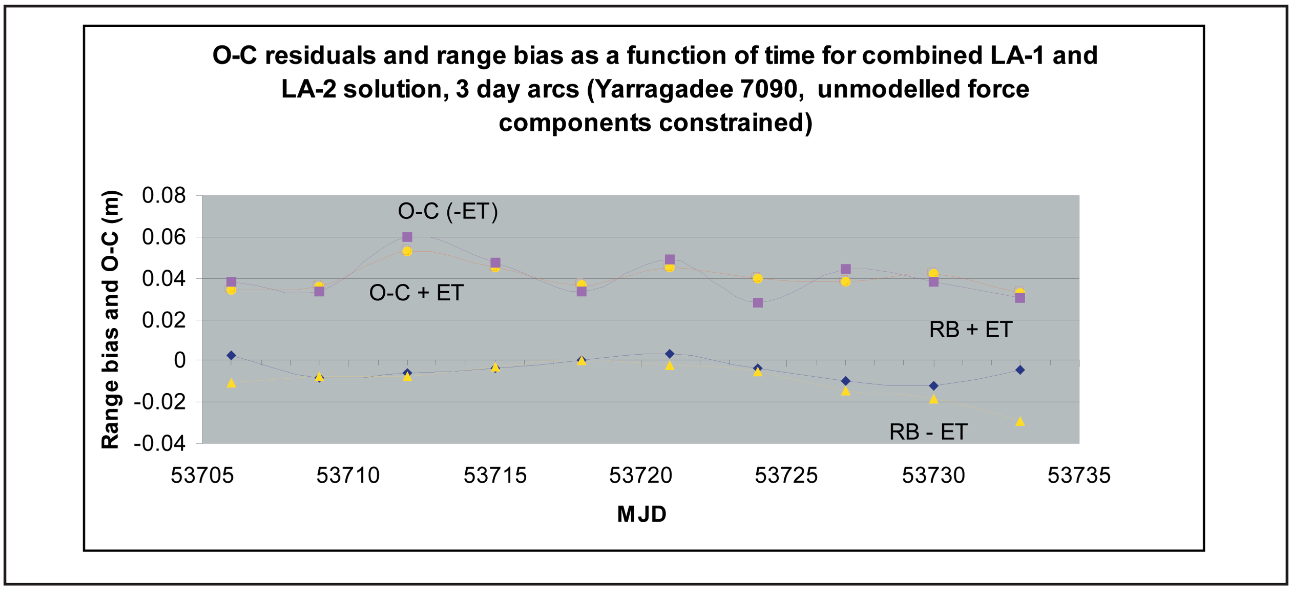

Figure 3. Range bias for the combined solution of LAGEOS-1 and LAGEOS-2 indicating a reduction in range bias as a result of including Earth-tide modelling. The unmodelled acceleration component was constrained and perturbations due to Earth-tide effects on the static gravity field were enabled.

orientations and surface temperatures), which can have the form (Montenbruck and Gill, 2000)

$$\ddot{\mathbf{r}} = \mathbf{E}(\mathbf{a}_0 + \mathbf{a}_1 \sin \nu + \mathbf{a}_2 \cos \nu)$$

This mismodelling occurs mainly at a frequency of one-cycle-per-revolution (1CPR). In the equation $\mathbf{a}_0$ is a constant acceleration bias, with $\mathbf{a}_1$ and $\mathbf{a}_2$ being the 1CPR coefficients and the true anomaly.

In our application, if this parameter is enabled, comparison between Earth-tide on or off becomes very difficult, as the solve-for parameter tends to absorb the additional range bias if Earth-tide corrections are disabled. When set to have very small a-priori estimated error, this solve-for parameter then becomes a consider parameter, as it comprises force model parameters that are supposed to be uncertain but are not modelled in the least-squares estimation. We assume that other software, when estimating the effect of including Earth-tide or when comparing between Earth-tide models will have to give careful attention to the effect that empirical accelerations have on the orbit estimation. A better fit to the data in the least-squares sense may not indicate that an improvement in orbit has been found as the empirical acceleration coefficients will adjust throughout the orbital fit process, leading to reduced range bias and O-C residuals. All other settings in our tests remained the same. Increasing the length from a short-arc (an orbital revolution to three days) to a long-arc solution (> three days), in general increases the errors in the calculated

Table 2. Summary of results listing the mean of the RMS values of the O-C residuals of three-day arcs and the mean of the range biases.

Solution (Earth-tide variation of gravity field disabled)	O-C (mean of RMS)	Mean range bias
LAGEOS-1 plus LAGEOS-2 (Earth-tide on)	0.085 ± 0.0088	0.014 ± 0.0103
LAGEOS-1 plus LAGEOS-2 (Earth-tide off)	0.100 ± 0.0144	0.035 ± 0.0078
LAGEOS-1 (Earth-tide on)	0.051 ± 0.0082	0.007 ± 0.0057
LAGEOS-1 (Earth-tide off)	0.040 ± 0.0046	0.006 ± 0.0054
LAGEOS-2 (Earth-tide on)	0.037 ± 0.0033	0.004 ± 0.0055
LAGEOS-2 (Earth-tide off)	0.047 ± 0.0081	-0.005 ± 0.0046
Solution (Earth-tide variation of gravity field enabled, unmodelled accelerations constrained)	**O-C (mean of RMS)**	**Mean range bias**
LAGEOS-1 plus LAGEOS-2 (Earth-tide on)	0.040 ± 0.0062	-0.004 ± 0.0053
LAGEOS-1 plus LAGEOS-2 (Earth-tide off)	0.040 ± 0.0053	-0.010 ± 0.0089
LAGEOS-1 (Earth-tide on)	0.035 ± 0.0081	-0.005 ± 0.0056
LAGEOS-1 (Earth-tide off)	0.035 ± 0.0081	-0.003 ± 0.0062
LAGEOS-2 (Earth-tide on)	0.048 ± 0.0178	0.002 ± 0.0187
LAGEOS-2 (Earth-tide off)	0.041 ± 0.0139	0.0005 ± 0.011

orbit as the modelled component reduces in accuracy due to several factors. Firstly, the software only estimates one solar reflection coefficient per solution. If more were estimated (say one for every six hours) unmodelled effects in satellite acceleration would be reduced. Secondly, all modelled accelerations including those resulting from the Earth's gravity field and perturbations due to the Sun, moon and planets are not perfect, therefore the errors increase and accumulate as a function of time. In addition, each solution is of course also affected by the quantity and quality of data available.

Results and discussion

For comparison purposes we initially disabled the additional acceleration caused by Earth-tide of the satellites due to fluctuations of the static gravity field caused by the solid Earth-tide and loosened the constraint on the a-priori error of the unmodelled force components (Table 2, first part). Enabling (Table 2, second part) the Earth-tide effect on the Earth's gravity field and constraining unmodelled perturbation modelling improves the range bias and O-C residuals substantially. This allows evaluation of both corrections; solid Earth-tide correction to station position and perturbation modelling of the satellite orbit due to variations in the static gravity field due to solid Earth-tide.

As can be seen in the first part of Table 2, results for the combined solution (LAGEOS-1 plus LAGEOS-2) were basically the same as for the independent orbital solutions. In general it was found that including the Earth-tide modelling and thus adding the Earth-tide amplitude vector to the SLR station coordinates improved the RMS values of the O minus C residuals and reduced the range bias. Reduction of the range bias after correcting for Earth-tide is an important result as it indicates correct functioning of the analysis program and the implementation of the position displacement vector resulting from the solid Earth-tide. These results are summarised in Table 2. All values are in metres. The separate solution for LAGEOS-1 however indicates an increase in the mean value of the O minus C RMS values when Earth-tide modelling is included. Inspection of the O minus C results for LAGEOS-2 when processed independently indicates several high O minus C values which affect the results adversely from what is expected, however there was no reason to remove these outliers. Figure 1 illustrates the results of including Earth-tide modelling in the combined satellite solution. The centre of each three-day period is taken as the epoch of the period and the time is indicated in Modified Julian Day (MJD). Range bias will be larger if some systematic effect exists (instrumental or otherwise) which makes the range to the satellite apparently greater or smaller.

Earth-tide at the Yarragadee site has a minimum amplitude of 2 cm and a maximum of about 16 cm for the period under investigation (see Figure 2). This will lead to a bias range modulated with the period of the Earth-tide if not corrected. Figure 2 depicts the station displacement vector due to Earth-tide for the period of data processing. If one compares this figure with the range bias (including Earth-tide) of Figure 3, there is an obvious correlation (0.64). The computed range after correction of the solid Earth-tide seems to be slightly biased towards being too large as the bias is more negative than positive; this could be an indication of over adjustment in the Earth-tide model.

Considering the second part of Table 2 there is a significant impact whether one includes the modelling of the effect of the Earth-tide on the static gravity field or not. It is clear that for best results both Earth-tide effects on the station position and the gravitational field of the Earth need to be included. The very small range bias in the test for a combined LAGEOS-1 and LAGEOS-2 solution where Earth-tide is enabled (-0.0040±0.0053 m) or disabled (-0.010±0.0089 m) indicates a statistical improvement in the average range bias, however, these very low range bias levels are already moving into the noise floor of the measurements (single normal point accuracy of 1 to 2 cm). The individual solutions for each of the satellites basically exhibit statistical noise. It is quite clear that enabling the additional acceleration of the satellite due to gravity field variations improves the O-C residuals by a factor of two. Figure 3 indicates a smoother O-C residual when Earth-tide is enabled and the range-bias hugs the zero line whereas without Earth-tide modelling the range bias tends to drift away. These range biases at this level are very close or within the statistical noise and does not indicate real instrumental biases. If there were a large (>1 cm) instrumental bias it would however be detected clearly.

Much longer data periods are available for both satellites and it is expected that processing several years of data will provide better results. We are currently coding the corrections due to atmospheric loading and these inclusions will also add to the total accuracy of the modelling.

Summary and Conclusions

The analysis of SLR data requires adequate modelling of the orbit via an orbit integrator that includes modelling of gravitational and non-gravitational forces perturbing the orbit of the satellite. In order to adjust the range as determined by the SLR system, corrections due to physical effects such as caused by the solid Earth-tide and the effects of the Earth-tide on the static gravity field need to be made. In the process of developing this SLR analysis software we have included Earth-tide modelling and have conclusively shown that the O-C residuals for LAGEOS-1 and LAGEOS-2 orbits are reduce from 8 cm to 4 cm for three-day arcs and that range bias is reduced as well. We also find that it is important to carefully evaluate the effect of unmodelled forces (*e.g.*1 CPR) modelling on the orbit and other solve-for parameters such as range biases. This will invariably be important for inter-comparison of differential Earth-tide and other loading models.

A comparison between the Yarragadee station position perturbation vector resulting from solid Earth-tide and calculated SLR range bias indicates a correlation, which probably results from an overestimate of the Earth-tide vector, as the range bias is predominantly negative. This sensitivity of the SLR technique indicates that it would be possible to test different Earth-tide models and assess them (or improve them) in terms of accuracy, leading to tuned station displacement due to solid Earth-tide.

Further improvements such as ocean-tide induced static gravity field variations and atmospheric-tide modelling will bring additional accuracy. Following these improvements, further work will concentrate on using SLR data to determine SLR station velocity vectors that have application in global plate tectonics and crustal dynamics studies.

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